JOURNAL OF AIRCRAFT
Vol. 29, No. 5, Sept.–Oct. 1992

Application of Advanced Multidisciplinary Analysis and Optimization Methods to Vehicle Design Synthesis

Robert David Consoli*
General Dynamics, Fort Worth, Texas 76012
and
Jaroslaw Sobieszczanski-Sobieski†
NASA Langley Research Center, Hampton, Virginia 23665

Advanced multidisciplinary analysis and optimization methods; namely, system sensitivity analysis and nonhierarchical system decomposition, are applied to reduce the cost and improve the visibility of an automated vehicle design synthesis process. This process is inherently complex due to the large number of functional disciplines and associated interdisciplinary couplings. Recent developments in system sensitivity analysis as applied to complex nonhierarchic multidisciplinary design optimization problems enable the decomposition of these complex interactions into subprocesses that can be evaluated in parallel. The application of these techniques may result in very significant computational and calendar time savings, furthermore they may result in significant cost, accuracy, and visibility benefits for the entire design synthesis process.

Introduction

AS in any engineering design, the vehicle design process has a qualitative side that depends on human intuition, creativity, judgment, and a quantitative side that must be supported by analysis being performed by computers. The entire vehicle design process is a very large undertaking, involving many engineers who tend to group according to their specialties. This grouping provides a natural basis for developing a broad framework to bring as much concurrent manpower as possible to bear on the project for compression of calendar time. On the quantitative side of the process, that mode of operation calls for decomposition of the entire problem into smaller subproblems that can be identified with the specialty groups. Numerous decomposition schemes have been proposed in the field of operations research, as surveyed in Ref. 1, and in the field of engineering applications, e.g., Ref. 2. In the latter reference, a distinction is drawn between a hierarchic decomposition in which there are several levels of subproblems, each depending on the input from a level above, and a nonhierarchic decomposition in which the subproblems are all interconnected at the same level.

In the hierarchic decomposition, each subproblem contains its own optimization. An example of an application to a large problem is detailed in Ref. 3. In the nonhierarchic decomposition, the optimization is performed at the system level for the entire vehicle. However, the sensitivity analysis that generates the derivatives needed for that optimization is based on many sensitivity analyses, each executed concurrently within specialty groups. Because in large system application, the sensitivity analysis may account for more than 90% of the time needed for the overall optimization, the concurrency of the discipline sensitivity analysis has a potential for compression of the overall time schedule. An even more important benefit of this approach is that it channels the quantitative information exchange among the specialists into the format of the derivatives, providing visibility of the mutual influences among the engineering groups. Experience with this approach has been accumulating recently as evidenced in Refs. 4 and 5.

The purpose of this paper is to show that system sensitivity analysis and nonhierarchical system decomposition can be an effective tool for configuration optimization when applied to an automated vehicle design synthesis process. To this end, the paper defines the need for a multidisciplinary approach to vehicle design synthesis, describes such an approach that has been adopted in industry, examines the role of sensitivity information in the design process, outlines an algorithm for system sensitivity analysis, and describes how a particular vehicle configuration optimization problem may be formulated on the basis of the system sensitivity data. The report concludes with a discussion of the numerical results of an example application.

Need for a Multidisciplinary Approach

Recent developments in the quantitative aspects of design; namely, advancements in both computer technology and the availability of sophisticated analysis and optimization methodologies, have resulted in quantum improvements in the computational environment that supports the design process. These developments have also served to improve the accuracy of the results obtained from this process by increasing both the number of participating functional disciplines and their contributing levels of detail. However, the increased number of functional disciplines involved in today's vehicle design trade studies has placed an even greater burden on the design process simply because of the increased number of interdisciplinary couplings that must be resolved.

A recent example of this complexity can be found in today's hypersonic vehicle design trade studies. In the case of hypersonic vehicle design and analysis, attempts to reduce the aerodynamic drag on the vehicle by elongating the forebody to a more slender shape also has an impact on the position of the nose shock relative to the inlet, which strongly influences the propulsive efficiency. Although this lengthening of the forebody is favorable in terms of reducing the aerodynamic drag on the vehicle, it may also reduce the vehicle's propulsive efficiency through the nose shock position effect. The net result may be an increase in the mission-sized takeoff

Presented as Paper 90-2.3.4 at the 17th Congress of the International Council of the Aeronautical Sciences, Stockholm, Sweden, Sept. 9–14, 1990; received Nov. 13, 1990, revision received July 5, 1991; accepted for publication Aug. 5, 1991. Copyright © 1990 by General Dynamics Corporation. All rights reserved. Published by the American Institute of Aeronautics and Astronautics, Inc., with permission.

*Senior Engineer, Fort Worth Division. Member AIAA.
†Head, Interdisciplinary Research Office. Associate Fellow AIAA.

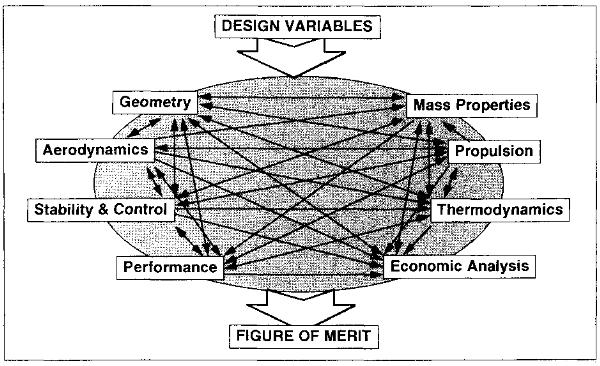

Fig. 1 Functional discipline interactions in design trade studies.

gross weight of the vehicle, which is in opposition to the result intended. Such aerodynamic-propulsion interaction is but one example of numerous complex functional discipline interactions commonly found in today's design trade studies. This is depicted by the arrows connecting the functional disciplines in Fig. 1.

Conventional approaches to resolving these interdisciplinary interactions usually involve performing parametric or one-factor-at-a-time analysis of the design variables of interest. This approach may also be prohibitively expensive and time consuming when applied to complex and computationally intensive design trade studies as detailed in Ref. 6. Often the result is to reduce the number of design variables of interest and to simplify or omit many of these interactions. Automating this process helps to bring the collective functional disciplines together by provides little benefit in reducing the computational time required to perform the trade study. Fortunately, the union and application of advanced multidisciplinary analysis and optimization methods to such a program can provide a viable and cost effective alternative to conventional approach.

Vehicle Design Synthesis Process—Description

Traditional vehicle design synthesis programs consist of, as a minimum, functional modules for geometry, aerodynamics, propulsion, mass properties, and performance. Depending on the level of detail required, functional discipline modules for economic analysis, structures, thermodynamics, stability and control, reliability/maintainability/supportability (RM&S), and avionics may also be available. In a multidisciplinary environment, each module is developed and maintained by its respective functional discipline and is updated accordingly, as new technology becomes available.

Such programs are usually self-contained and provide the capability to perform vehicle design trade studies rapidly. A typical organization of primary functional discipline modules and overall description of the design synthesis process is illustrated in Fig. 2. The design synthesis process begins with the definition of the baseline vehicle concept and the baseline mission profile from which the baseline vehicle derives. The baseline mission profile provides the information needed by the performance module to compute the fuel required for the mission. Once the baseline information has been loaded into the synthesis program, each functional discipline module is called in a sequential manner. This process begins with the geometry module, which computes the geometric characteristics of the vehicle in addition to the fuel available for the mission. The mass properties module is then called to compute the scaled and unscaled component weights for the vehicle. Next, the aerodynamic and propulsion modules are called to generate tabular data for use in the mission calculations. Finally, detailed mission performance data are computed, which results in the fuel required.

Next, a comparison is made of the fuel available and the fuel required. If there is an excess amount of fuel available, the vehicle is sized down (according to a variety of vehicle

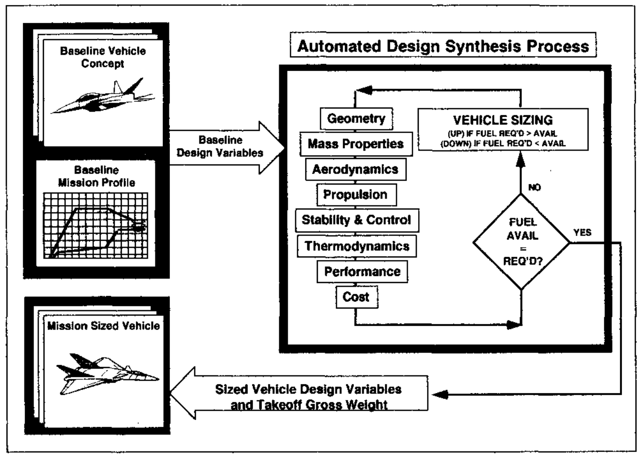

Fig. 2 Automated design synthesis functional module organization and process description.

scaling methods, dependent on the configuration type). If there is a fuel deficit, the vehicle is sized up to accommodate the required amount of fuel. This iterative process, which typically converges in three to four iterations, results in a mission-sized takeoff gross weight for the input set of design variables. In the case of design variable optimization, the fuel balance needed to size the vehicle is formulated as a design constraint and added to any user-specified performance requirements. The optimization algorithm then predicts new values for the design variables that improve the objective function and promote constraint satisfaction.

The optimization process requires information from the design synthesis process in order to minimize the user-specified objective function, which is mission-sized takeoff gross weight or total life cycle cost. Typically, this information is provided in the form of total derivatives for the objective function with respect to each of the specified design variables. The next section of this paper describes how system sensitivity analysis can be used to provide the necessary design information to the optimizer.

System Sensitivity Analysis—An Outline

The numerical side of design process depends on the analysis of a mathematical model of the design artifact, in this case an aircraft or vehicle. Analysis of the mathematical model accepts an input that describes the environment within which the vehicle operates and the stimuli to which it responds. This yields an output vector of the data on the ensuing behavior (response) of the vehicle. These data are valid for a particular set of values of the design variables. It is essential for the designer to know how that behavior will change if any of the design variables are altered. Under the prevailing practice, answering such "what if" questions requires the perturbation of one variable at a time and the repetition of the analysis in order to generate information for a finite differencing or for a parametric study. *In other words, one needs to test the effect of a particular change under consideration in order to determine whether that change is going to have the desired effect.*

This trial-and-error approach is a consequence of working with zero-order information; that is, the function values for the given arguments. If we also had the first-order information and the values of the derivatives of those values with respect to design variables, the effect of any comtemplated change of the variables could be evaluated before committing to further implementation and reanalysis. The purpose of the system sensitivity analysis is to produce such first-order information more efficiently and accurately than can be obtained by finite differencing. Mathematical means of producing first-order information are tantamount to giving the designers a mathematical model of the design for quick answers to "what if" questions, as is often the case in conceptual design.

For large and complex systems, the perturb-and-reanalyze approach to generating data for finite differences that approximate the derivatives may be impractical, as well as inaccurate and too costly. However, an algorithm for the system sensitivity analysis was introduced in Ref. 7 as an alternative to finite differencing in the generation of first order derivatives. This algorithm was extended to include higher order derivatives in Ref. 8. An example of the algorithm's application, including the use of the first-and-second order derivatives in optimization, is given in Ref. 9. Although the references provide a comprehensive description of the algorithm, its outline is given herein to make this paper self-contained.

System Sensitivity Approach

The system sensitivity algorithm, developed in Ref. 7, treats the system as an assemblage of black boxes, with each box containing a mathematical model to represent a particular aspect of the system behavior or a physical subsystem. It is not important to describe the contents of each black box as long as the inputs and outputs for each block of boxes are defined. Each of the black boxes is coupled because output from one black box may be transmitted as input to another. An ith black box is viewed as a converter that transforms the design variables, denoted by input vector $\boldsymbol{X}$, and the outputs from the other boxes, denoted by input vectors $\boldsymbol{Y}_j$, to an output vector $\boldsymbol{Y}_i$, assuming that $j \neq i$. Of course, it is recognized that not every element of $\boldsymbol{X}$ and $\boldsymbol{Y}_j$ is actually used. The usage is selective because not all output variables from each black box are inputs into the other black boxes.

If the input-to-output conversion for each black box produces $\boldsymbol{Y}_i = f(\boldsymbol{X}, \boldsymbol{Y}_j)$ as a continuous function of the arguments $\boldsymbol{X}$ and $\boldsymbol{Y}_j$, the partial derivatives $\partial Y_i/\partial X$ and $\partial Y_i/\partial Y_j$ exist. They represent the direct influence of these arguments on the output as opposed to an indirect influence. For example, the sweep angle affects the pressure distribution on the wing directly through aerodynamic predictions and indirectly through its influence on the structural analysis output of displacements.

The partial derivatives may be computed by finite differencing or by a quasianalytical technique. The former is the simplest to implement without accessing the inner workings of a black box; the latter is inherently more efficient and more accurate but requires modifications to the black box analysis. Quasianalytical methods for sensitivity analysis have been well established in some disciplines, notably in structures (a survey was given in Ref. 10). Initiation of similar developments in CFD is discussed in Ref. 11 and in its corresponding references.

Regardless of the method used, a black box sensitivity analysis is a local operation applied to a black box isolated from all others. Therefore, all such analyses may be executed concurrently—an attractive feature considering the availability of distributed computing. Other important features not to be overlooked are the visibility into the functional discipline interactions that are provided by the partial derivatives and the design sensitivity information provided by the total derivatives. This information, which is not traditionally available using a one-factor-a-time approach to vehicle design synthesis, can be useful in determining what impact a proposed design change will have on the output from each black box as well as for the overall system objective.

The system sensitivity approach seems ideally suited for implementation into an automated vehicle design synthesis program because the functional modules required to compute the partial derivatives are readily available. The benefits of this approach are the reduction in the time and cost associated with performing design trade studies and the improved visibility provided by the derivatives. This is particularly true when utilizing a design synthesis program in an optimization mode because the total derivatives obtained by solving the Global Sensitivity Equations (GSE) developed in Ref. 4, can also be used to drive the optimization.

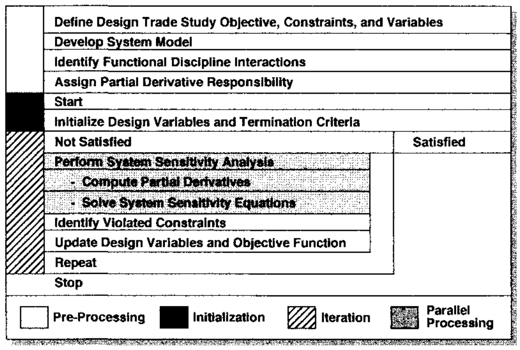

Fig. 3 Chapin-format of optimization procedure using the GSE.

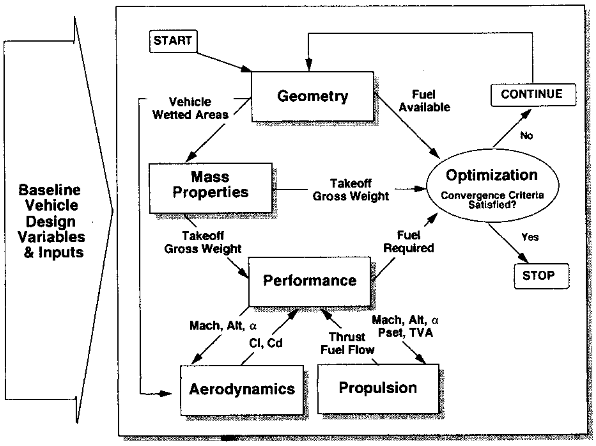

Fig. 4 System model for example application.

Figure 3 contains a flowchart (Chapin format) that summarizes the sequence of steps that should be taken in formulating and solving the GSE. As indicated in Fig. 3, the steps required to formulate and solve the GSE can be separated into four categories. These categories are preprocessing, initialization, iteration, and parallel processing. The first four steps are included in the preprocessing category because they involve the basic formulation of the problem. Step 5 is included as part of the intialization process. Steps 6–8 involve computation of the partial derivatives, the solution of the GSE, and the overall optimization of the problem, which is inherently an iterative process. The final category, parallel processing, is unique to step 6 and involves the concurrent computation of the partial derivatives, which reduces the overall computation time for this process considerably.

Thus far, this paper has defined the need for a multidisciplinary approach to vehicle design synthesis, described automated vehicle design synthesis in generic terms, introduced the system sensitivity approach, and summarized the steps required to formulate and solve the GSE. The next section of the paper describes in detail the application and implementation of the system sensitivity approach to an automated design synthesis program.

GSE Application to Design Synthesis Process

Using the procedure outlined in Fig. 3, a system model for the primary functional discipline modules was developed. This system model, displayed in Fig. 4, identifies the relevant functional disciplines and associated interactions included in this application.

The primary functional disciplines of interest for this study are geometry, mass properties, performance, aerodynamics, and propulsion. Each of the disciplines is represented in Fig. 4 by black boxes (i.e., shadowed boxes in the fig.). Experience and familiarity have shown that the calculations for the performance module are the most computationally intensive. The aerodynamic module and the propulsion module calculations

Table 1 Functional module description for example application

Module	Inputs	Outputs	Design variables	Constants
Geometry	None	Fuel available vehicle wetted areas	All	Baseline geometry information
Mass properties	Fuel available vehicle wetted areas	Takeoff gross weight empty weight	All	Mass properties calibration factors
Performance	Takeoff gross weight empty weight	Fuel required performance constraints	All	Baseline mission rules
Aerodynamics	Mach number altitude angle of attack	Lift coefficient drag coefficient	All aerodynamic related	Aerodynamics calibration factors
Propulsion	Mach number altitude angle-of-attack power setting thrust vector angle	Net thrust fuel flow	All propulsion related	Propulsion calibration factors

are less intensive primarily because of the calibration process employed in the program and because they are at a level of analysis characteristic of the conceptual design stage. The geometry module and the mass properties module calculations are the least intensive. The optimization module checks whether the convergence criteria, which includes the fuel balance and performance constraints, have been satisfied. If not, the module predicts new values for the design variables that improve the objective function and continues sequentially through the system again.

Table 1 contains a brief description of the inputs (including the design variables), outputs, and constants for each of the functional module included in this example. Using the numbering scheme provided in Table 1 and the system model displayed in Figure 4, the corresponding GSE for this system in Jacobian matrix form is represented by Eq. (1):

$$\begin{bmatrix} I & 0 & 0 & 0 & 0 \\ -J_{21} & I & 0 & 0 & 0 \\ 0 & -J_{32} & I & -J_{34} & J_{35} \\ -J_{41} & 0 & -J_{43} & I & 0 \\ 0 & 0 & -J_{53} & 0 & I \end{bmatrix} \begin{Bmatrix} \dfrac{dY_1}{dX} \\ \dfrac{dY_2}{dX} \\ \dfrac{dY_3}{dX} \\ \dfrac{dY_4}{dX} \\ \dfrac{dY_5}{dX} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial Y_1}{\partial X} \\ \dfrac{\partial Y_2}{\partial X} \\ \dfrac{\partial Y_3}{\partial X} \\ \dfrac{\partial Y_4}{\partial X} \\ \dfrac{\partial y_5}{\partial X} \end{Bmatrix} \tag{1}$$

The actual implementation of the GSE in this application differed slightly from that of Fig. 4 primarily due to the following considerations of computational efficiency. As explained in Ref. 7, the computational cost of the system sensitivity analysis critically depends on the number of elements in the vectors Y_j (called the interaction band width) transmitted from one black box to another. This is especially true when the partial derivatives are computed by finite differencing, as is the case in the problem at hand, because the number of executions of the analysis in the ith black box depends on the number of partial derivatives that need to be calculated for that black box.

In the system depicted in Fig. 4 and defined in Table 1, the interaction band width is particularly wide between the performance, aerodynamics, and propulsion black boxes owing to the large Mach number and altitude coordinates that define a given mission profile. On the other hand, the band width between performance, geometry, and mass properties black boxes is very narrow. Taking advantage of that narrow bandwidth, the number of executions of the performance, aerodynamics, and propulsion modules were drastically reduced by merging aerodynamics and propulsion black boxes into the performance black box, as shown in Fig. 5.

Using the notation established in Table 1 and considering that the performance black box actually represents the combined effect of performance, aerodynamics, and propulsion, the resulting GSE for this system is given by Eq. (2)

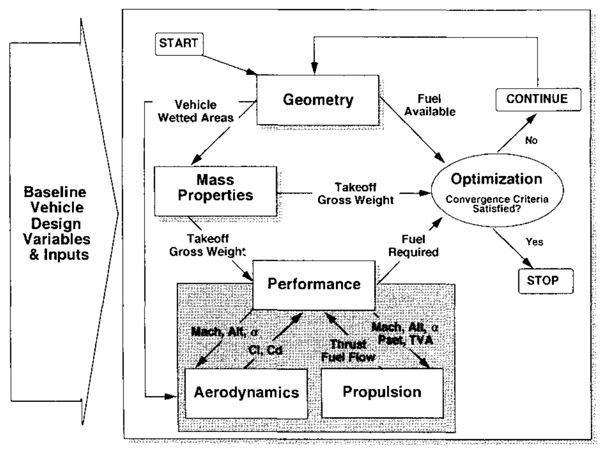

Fig. 5 Revised system model for example application.

$$\begin{bmatrix} I & 0 & 0 \\ -\dfrac{\partial \mathrm{Mass}}{\partial \mathrm{Geom}} & I & 0 \\ -\dfrac{\partial \mathrm{Perf}}{\partial \mathrm{Geom}} & -\dfrac{\partial \mathrm{Perf}}{\partial \mathrm{Mass}} & I \end{bmatrix} \begin{Bmatrix} \dfrac{d\mathrm{Geom}}{dX} \\ \dfrac{d\mathrm{Mass}}{dX} \\ \dfrac{d\mathrm{Perf}}{dX} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial \mathrm{Geom}}{\partial X} \\ \dfrac{\partial \mathrm{Mass}}{\partial X} \\ \dfrac{\partial \mathrm{Perf}}{\partial X} \end{Bmatrix} \tag{2}$$

Note that the system is still nonhierarchical due to the lateral linking between the mass properties and performance black boxes. The number of partial derivative calculations has been reduced from approximately several thousand to less than one hundred. The system model and associated functional module interactions, as displayed in Fig. 4, were incorporated into the example vehicle design synthesis program. The next section describes the results obtained for an example application using this system model and associated GSE.

Example Application

The synthesis of a hypersonic interceptor vehicle was accomplished using the system sensitivity information provided by the GSE's formulated previously. The example of configuration is a Mach 5.5 vehicle capable of intercepting adversary aircraft at high speeds in minimum time. The baseline mission profile for this application is displayed in Fig. 6.

The explicit mission rules that define the policy, propulsion power-setting, and termination criteria for each mission segment are presented in Table 2. The mission rules that are required inputs for the performance module are presented here to illustrate the complexity of this application's mission profile and to demonstrate the versatility of this module.

The mission rules do not fully define the mission profile because the resulting segment time, fuel-used, and range are

Table 2 Baseline mission rules for example application

Segment	Policy	Power setting	Termination
Takeoff	Fixed power	Max power turbojet	Alt = 100 Mach = 0.5
Climb	Min time	Max power: turbojet to 0.8M turboramjet to 3.25M ramjet to 5.5M	Cruise altitude
Cruise	Optimum altitude Constant mach = 5.5	Ramjet power as required	Search for cruise distance
Descent	L/D max	Idle power ramjet	Alt = 50000 Mach 3.25
Turn	Sustained structural load factor = 2.5	Ramjet power as required	180 degree turn
Climb	Constant mach = 3.25	Max power ramjet	Cruise altitude
Cruise	Optimum altitude Constant mach = 3.25	Ramjet power as required	Search for cruise distance
Descent	L/D max	Idle power turboramjet	Mach = 0.8
Landing	Fixed power	Idle power turbojet	Alt = 0.0 Mach = 0.0

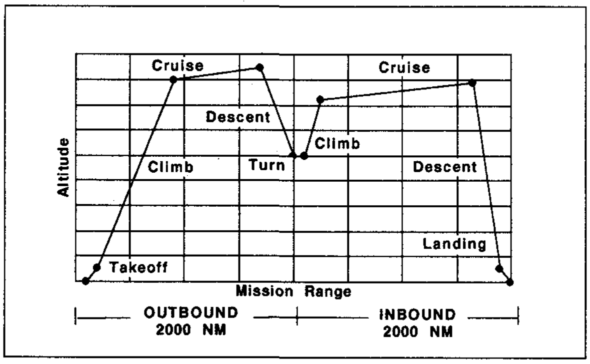

Fig. 6 Baseline mission profile for example application.

a direct function of the vehicle's design variables and will change with each iteration of the optimizer. For instance, the minimum time trajectory for the climb segment is a function of the wing area and engine size of the vehicle. A change in either of these design variables directly influences the computed trajectory, affecting the final vehicle weight for the segment, which, in turn, affects the optimum cruise altitude. However, the baseline mission profile stipulates that the outbound and inbound range for this mission profile must be 2000 nautical miles. This distance defines the adversary interception point for this application and also indirectly defines the termination range for each cruise segment.

The design variables were limited to wing area, turbojet size, and ramjet size. The engine-scale factors were of significant interest because the powerplant is a methylcyclohexane (MCH)-fueled turbo-ramjet and the engine cycles operate independently or in unison depending on the particular Mach regime. When the turbojet operates independently, the ramjet airflow is ducted. Similarly, when the ramjet operates independently, the turbojet inlet is closed. Due to structural and thermodynamic considerations, it was assumed that the fuerl required to perform the specified mission was stored in the fuselage. These considerations made it necessary to include another design variable, fuselage length, in the formulation of this problem. Variation of the fuselage length served to define the amount of fuel available to perform the specified mission.

Optimization Procedure

The objective of the example design synthesis trade study was to reduce the mission-sized takeoff gross weight by manipulating the design variables (wing area, turbojet size, ramjet size, *and* fuselage length), subject to the following constraints:

$$1)\ |\text{Fuel}_{\text{REQUIRED}} - \text{Fuel}_{\text{AVAILABLE}}| \leq 0.1 \quad (3)$$

$$2)\ \text{Time-to-Station} \leq 55\ \text{min} \quad (4)$$

$$3)\ \text{Takeoff Velocity} \leq 195\ \text{kt} \quad (5)$$

It should be emphasized that by including the fuel available vs fuel required as a constraint, the optimization procedure in Fig. 5 is employed in a dual role. It synthesizes a flyable vehicle and simultaneously configures that vehicle toward an optimum. This eliminated the need to achieve a fuel balance prior to performing the optimization and reduced the computational time required to locate the optimum design variables significantly. In conventional practice, these two functions would be performed separately and consecutively.

The usual practice of applying optimization methods to large-scale problems involves coupling the optimizer with approximate analysis; for example, a linear extrapolation based on the sensitivity derivatives instead of the full analysis.[12] This approach has been widely accepted in industry primarily because of the computational time savings that result when compared to coupling the optimizer directly with the full analysis. In formulating a general scheme for the incorporation of optimization methods for this application, it was determined that use of approximate analysis was not necessary. This determination was based on the relatively small number of design variables common in conceptual design trade studies and the fact that the time required for a function evaluation is not computationally prohibitive. For this reason, the optimizer was coupled directly with the full analysis methods as indicated in Fig. 5.

This optimization problem was solved using the aforementioned system model in conjunction with the ADS optimization program, described in detail in Ref. 13. ADS is a general purpose numerical optimizaton program, which contains a variety of algorithms. The algorithm selected for this particular problem was sequential linear programming (SLP) using the modified method of feasible directions optimization strategy. ADS is well suited for solving problems formulated using the GSE approach because the program can accept user-supplied gradient information readily.

Application Results

The optimum design variables were obtained by solving the GSE simultaneously for the total derivatives of the objective function and constraints with respect to each of the design variables for each optimization iteration and by providing this information to the ADS optimizer. Table 3 displays the optimum values for the design variables (normalized with respect to baseline values), constraints, and objective function for this example application. In this table, results obtained parametrically using traditional methods are contrasted with results obtained from using the GSE approach with the results

Table 3 Optimization results for example application

	Baseline value (initial)	Traditional method results	GSE method results
Optimization parameter			
Wing area[a]	1.0000	0.7905	0.7898
Turbojet size[a]	1.0000	1.2342	1.2346
Ramjet size[a]	1.0000	0.8001	0.8014
Fuselage length[a]	1.0000	0.9587	0.9585
Objective function			
Takeoff gross weight[a]	1.0000	0.8786	0.8734
Constraint functions			
|Fuel balance|[a] <0.1	1.0000	0.0325	0.0058
Intercept time <55 min	62.345	52.691	52.345
Takeoff velocity <195 knots	216.74	191.32	191.06

[a]Normalized with respect to baseline value.

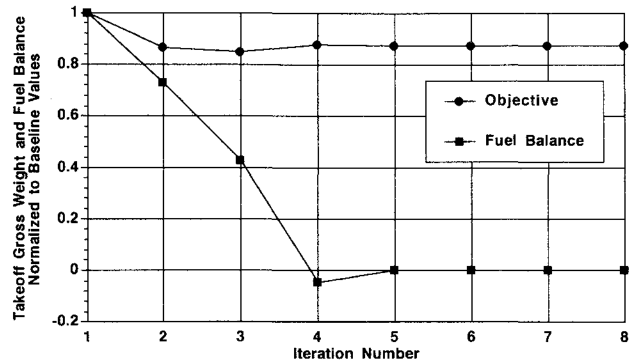

Fig. 7 Sizing history plots for example application.

obtained from applying the ADS optimizer. Note that the results obtained from using the optimizer are in complete agreement with the traditional approach.

In performing the optimization, the overall mission-sized takeoff gross weight of the vehicle was reduced approximately 13%, which included the satisfaction of the fuel balance and performance constraints. This was accomplished by reducing the wing area by 21%, reducing the ramjet size by 20%, increasing the turbojet size by 23%, and reducing the fuselage length by approximately 4%. Initially, the fuel balance and performance constraints, represented by Eqs. (3), (4), and (5), were violated. However, for the optimized vehicle, all the specified constraints were sufficiently satisfied. Figure 7 displays a sizing history plot for the objective function and fuel balance constraint. The figure displays takeoff gross weight and fuel balance (fuel available minus fuel required) fractions for each iteration.

Selection of the SQP optimization strategy for ADS seemed to converge quickly. Primarily, this is because gradient information for all the design variables was supplied simultaneously once the GSE were solved for the total derivatives. Normally, optimization algorithms calculate gradients internally one design variable at a time, which considerably increases the number of iterations required to locate the optimum design. An obvious benefit of this approach is the visibility provided by the partial and total derivatives. Table 4 displays a sampling of some of the partial derivatives obtained from this application.

The partial derivatives shown in the table are normalized with respect to baseline values and are expressed as a percent-to-percent basis. Partials such as these indicate the relative influence of each input variable to the output variables of interest, which can be of significant interest to the design team. If a partial derivative is negative, it means that a 1% increase in the input variable Y_j results in a corresponding decrease (measured in percent) in the output variable Y_i. Those

Table 4 Baseline partial derivatives for example application

Partial Derivative[a]	$\frac{\partial(Y_1)}{\partial X_i}$	$\frac{\partial(Y_2)}{\partial X_i}$	$\frac{\partial(Y_3)}{\partial X_i}$	$\frac{\partial(Y_4)}{\partial X_i}$
Baseline				
X_1 = Wing area	0.0000	0.2040	0.0676	−0.0644
X_2 = Turbojet size	0.0000	0.1044	0.0346	−0.0241
X_3 = Ramjet size	0.0000	0.3215	0.1065	−0.0462
X_4 = Fuselage length	3.0301	0.0012	0.0054	1.5604
Optimum				
X_1 = Wing area	0.0000	0.1871	0.0606	−0.0534
X_2 = Turbojet size	0.0000	0.1197	0.0387	−0.1317
X_3 = Ramjet size	0.0000	0.3213	0.1040	−0.0313
X_4 = Fuselage length	3.0301	0.0010	0.0049	0.9514

[a]Y_1 = Fuel available; Y_2 = Empty weight; Y_3 = Takeoff gross weight; Y_4 = Fuel required.

Table 5 Baseline total derivatives for example application

Total derivative[a]	$\frac{d(Y_1)}{dX_i}$	$\frac{d(Y_2)}{dX_i}$	$\frac{d(Y_3)}{dX_i}$	$\frac{d(Y_4)}{dX_i}$
Baseline				
X_1 = Wing area	0.0000	0.2040	0.0676	0.9557
X_2 = Turbojet size	0.0000	0.1044	0.0346	0.1618
X_3 = Ramjet size	0.0000	0.3571	0.1183	1.7760
X_4 = Fuselage length	3.0301	1.3411	2.4280	11.863
Optimum				
X_1 = Wing area	0.0000	0.1871	0.0606	−0.0432
X_2 = Turbojet size	0.0000	0.1197	0.0387	−0.1482
X_3 = Ramjet size	0.0000	0.3584	0.1160	−0.3147
X_4 = Fuselage length	3.0301	1.2978	2.4206	−0.7166

[a]y_1 = Fuel available; Y_2 = Empty weight; Y_3 = Takeoff gross weight; Y_4 = Fuel required.

partials specified as zero are simply not a function of the input variable Y_i. For this application, the assumption was made that the fuel required to perform the mission is stored in the fuselage. For this reason the patial derivative for fuel available as a function of wing area is zero, as are similar partial derivatives for turboject size and ramjet size.

It is also interesting to compare the partial derivatives computed for the baseline vehicle to those computed for the optimized configuration in order to measure the change in one of the parameters from the baseline to the sized vehicle. The relative magnitudes of each of the partial derivatives also provide information on how much influence each partial has on the overall objective function for each iteration. Based on the information provided in Table 4, fuselage length has the largest impact on fuel available, ramjet size has the largest impact on empty and takeoff gross weight, and fuselage length has the greatest impact on the fuel required on a percent-to-percent basis. Similar information is available for the total derivatives for which a sampling is displayed in Table 5.

The total derivatives are fundamentally different from the partial derivatives because they measure the influence of the design variables while fully accounting for the mutual couplings in the system. Similar to Table 4, the total derivatives in Table 5 show the infuence that a 1% increase in each of the design variables has on the output variable in the numerator, also expressed in percent. For those cases in which the total derivatives and partial derivatives are equal (as is the case for the fuel available partials in Tables 4 and 5), the output variables from the particular black box are uncoupled with inputs from the other black boxes. In particular, notice the relatively small change between the baseline total derivatives for takeoff gross weight with respect to a 1% change in fuselage length when compared to values at the optimum. In contrast, note the large reduction in the influence of a 1% change in fuselage length to fuel required for the baseline total derivatives when compared to values at the optimum. The changes in these total derivatives can be explained by evaluating the convergence criteria and associated constraints for the example application, which involved sizing the vehicle by varying the fuselage length until fuel available equaled fuel required.

The total derivatives for the objective and constraint functions provided the information required by the ADS optimization program to determine the proper search direction. The total derivatives of mission-sized takeoff gross weight with respect to each design variable provided the gradient information for the objective function. There seems to be very little change in this total derivative for the optimized vehicle when compared to the baseline vehicle. This is probably due to the relatively strong impact of the fuel balance constraints on the mission-sized takeoff gross weight for this problem.

A comment regarding the accuracy of the derivatives obtained from the GSE approach is in order. In general, the accuracy of the derivatives is a direct function of the incremental step size used when computing the partial derivatives by finite differencing. The accuracy may be increased by experimenting with the step size and by employing a multipoint difference scheme. However, this increased accuracy comes at a price of increased computational time. Throughout this paper, the incremental step size used to compute all the derivatives was 1%. This particular step size was selected based on experimentation that revealed that this increment would yield accurate results without a large penalty in computational time.

Application Cost and Benefits

Due to the development status of the example program, it was not possible to perform this particular application in a parallel processing environment. However, because the computational time required to execute each functional module is readily available, the resulting time savings for performing this design trade study in a parallel processing environment can be estimated using an approach discussed in Ref. 14. The time savings can be estimated by comparing the time required to perform the trade study using the GSE-based method in both a sequential and parallel processing environment to that required by the conventional approach.

However, this comparison would not be justifiable without considering the fact that the traditional method did not involve the application of optimization methods. Obviously, a percentage of the time savings that may be realized using the GSE approach is directly attributable to the application of an optimizer to the problem. For this reason, the computational time required to locate the optimum design variables for the example hypersonic application will be tabulated and contrasted for the folowing approaches: 1) traditional method consisting of a parametric surveying of the design space, followed by a one-factor-at-a-time variation of the design variables within the subspace of interest, without the use of formal optimization; 2) optimization method, which involved computing the total derivatives by finite differencing on the full analysis (Fig. 5), without the use of the GSE; 3a) using the GSE instead of the above finite differencing, executed on a single processor computer; and 3b) using the same approach described in 3a but employing a parallel processing computer.

Figure 8 displays a comparison of the CPU elapsed time for methods 1, 2, 3a, and 3b, normalized by the time required by method 1. For a single processor computer, the CPU elapsed time is equal to the CPU time, but for an "N" processor computer, the CPU elapsed time is compressed by using all the processors simultaneously to the largest extent possible (in the case at hand, $N = 12$). Application of method 2 reduces the time to 23%. Replacing method 2 with method 3a brings in the penalty for computing partial derivatives without, as yet, taking advantage of the parallelism intrinsic in the GSE method. Consequently, the time increases to 69%. However, employing method 3b, which takes advantage of such parallelism fully reduces the time to 4%—nearly one-sixth of the time required for method 2. The relative time

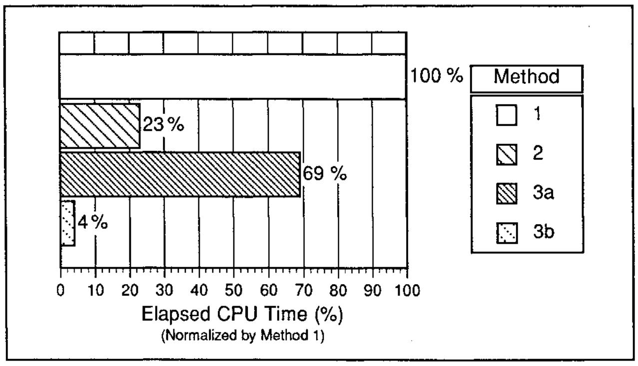

Fig. 8 Elapsed CPU time comparison for methods 1, 2, 3a, and 3b.

advantage of method 3b over method 2 depends on the size of the problem and the number of modules in the system whose partials may be computed concurrently. Therefore, that advantage is expected to increase with the size of the problem.

Nevertheless, the benefits of applying the GSE approach extend beyond the computational time savings that result in a parallel processing environment, because the GSE also provides the partial and total derivative visibility that methods 1 and 2 distinctly lack. Results from these comparisons reemphasize the need to consider the availability of parallel processing when applying the GSE approach to vehicle design synthesis trade studies. Even if concurrent computation of the partial derivatives is not feasible, benefits still accrue from the functional advantages associated with decomposing a complex system and assigning responsibility to the functional disciplines involved. In many cases, these benefits simply outweigh any computational disadvantages associated with the sequential processing of the partial derivatives. However, if parallel processing is available, then the visibility and computational benefits of the GSE approach can be directly realized.

Conclusions

The application of system sensitivity analysis and nonhierarchical system decomposition methods to an automated vehicle design synthesis program seems to be highly beneficial, particularly when parallel processng becomes an everyday reality. An example served to quantify several important features and benefits resulting from the union of these techniques applied to a vehicle design synthesis program. These features and benefits include the ability to: 1) respond directly to the ever-increasing complexity of the vehicle design synthesis process by realizing the interdisciplinary synergism; 2) encourage and support the use of concurrent engineering principals; 3) take advantage of technological advancements made in the areas of distributed and parallel processing; 4) address the qualitative aspects of the design process by supportig human judgment decisions and providing for a means of communication betwee disciplines; and 5) quantify the direct and indirect impact on system performance of a proposed design change accounting for all disciplines involved in the design synthesis process.

References

[1]Barthelemy, J. F., "Engineering Applications of Heuristic Multilevel Optimization Methods," *Second NASA/Air Force Symposium on Recent Advances in Multidisciplinary Analysis and Optimization*, NASA CP-3031, Hampton, VA, Sept. 28–30 1988, Pt. 3, pp. 1029–1038.

[2]Sobieszczanski-Sobieski, J., "Multidisciplinary Optimization for Engineering Systems: Achievements and Potential," *Lecture Notes in Engineering, DLR 47*, edited by H. W. Bergman, Springer-Verlag, Berlin, 1989.

[3]Wrenn, G. A., and Dovi, A. R., "Multilevel Decomposition Ap-

proach to the Preliminary Sizing of a Transport Aircraft Wing," *Journal of Aircraft*, Vol. 25, No. 7, 1988, pp. 632–638.

[4]Abdi, F. F., Ide, H., Levine, M., and Austel, L. G., "The Art of Spacecraft Design: A Multidisciplinary Challenge," *Second NASA/Air Force Symposium on Recent Advances in Multidisciplinary Analysis and Optimization*, NASA CP-3031, edited by J. F. Barthelemy, Hampton, VA, Sept. 28–30 1988, p. 1137.

[5]Grossman, B., Haftka, R. T., Kao, P. J., Polen, D. M., Rohani, M., and Sobieszczanski-Sobieski, J., "Integrated Aerodynamic-Structural Design of a Transport Wing," AIAA/AHS/ASEE Aircraft Design, Systems and Operations Conference, AIAA Paper 89-2129, Seattle, WA, July 31–Aug. 2, 1989.

[6]Sobieszczanski-Sobieski, J., "Sensitivity Analysis and Multidisciplinary Optimization for Aircraft Design: Recent Advances and Results," *International Council of the Aeronautical Sciences, 16th Congress*, Vol. 2, Jerusalem, Aug.–Sept. 1988, pp. 953–964.

[7]Sobieszczanki-Sobieski, J., "On the Sensitivity of Complex, Internally Coupled Systems," *AIAA Journal*, Vol. 28, No. 1, Jan. 1990, pp. 153–160; also published as AIAA/ASME/ASCE/AHS 29th Structures, Structural Dynamics and Materials Conference, AIAA Paper CP-88-2378, Williamsburg, VA, April 1988; and NASA TM 100537, Jan. 1988.

[8]Sobieszczanski-Sobieski, J., "Higher Order Sensitivity Analysis of Complex, Coupled Systems," *AIAA Journal*, Vol. 28, No. 4, 1990, pp. 756–758.

[9]Abdi, F. F., Ide, H. Shankar, V. J., and Sobieszczanski-Sobieski, J., "Optimization for Nonlinear Aeroelastic Tailoring Criteria," *International Council for the Aeronautical Sciences, 16th Congress*, Vol. 2, Jerusalem, Aug.–Sept. 1988, pp. 1083–1091.

[10]Adelman, H. A., and Haftka, R. T., "Sensitivity Analysis of Discrete Structural Systems," *AIAA Journal*, Vol. 24, No. 5, 1986, pp. 823–832.

[11]Yates, E. C. "Aerodynamic Sensitivities from Subsonic, Sonic, and Supersonic Unsteady, Nonplanar Lifting-Surface Theory," NASA TM 100502, Sept. 1987.

[12]Sobieszcanski-Sobieski, J., "Optimization in Aerospace Structures," *Engineering Design*, edited by R. R. Leveary, North-Holland, New York, 1988.

[13]Vanderplaats, G. N., "ADS—A FORTRAN program for Automated Design Synthesis," NASA CR 172460, 1984.

[14]Sobieszczanski-Sobieski, J., "On the Sensitivity of Complex Internally Coupled Systems," AIAA Paper 88-2378, April 1988.

Recommended Reading from the AIAA Education Series

Basic Helicopter Aerodynamics

J. Seddon

Basic Helicopter Aerodynamics introduces the theory of rotary-wing aircraft for undergraduate and graduate students. The author explains the analytical treatment and solutions of helicopter theory so that the reader may fully understand the physical phenomena. Many diagrams, drawings, graphs, and representative sets of data augment the text.

All of the topics necessary for a complete understanding of single-rotor helicopter aerodynamics are included: basic physical concepts for the helicopter rotor in vertical and forward flight, including momentum theory and wake analysis; blade element theory; aerodynamic design; performance; trim; static and dynamic stability; control; and autostabilization.

1990 133 pp., illus. Hardback • ISBN 0-930403-67-3
AIAA Members $39.95 • Nonmembers $49.95 • Order #: 67-3 (830)

Place your order today! Call 1-800/682-AIAA

AIAA

American Institute of Aeronautics and Astronautics
Publications Customer Service, 9 Jay Gould Ct., P.O. Box 753, Waldorf, MD 20604
Phone 301/645-5643, Dept. 415, FAX 301/843-0159

Sales Tax: CA residents, 8.25%; DC, 6%. For shipping and handling add $4.75 for 1-4 books (call for rates for higher quantities). Orders under $50.00 must be prepaid. Please allow 4 weeks for delivery. Prices are subject to change without notice. Returns will be accepted within 15 days.